ISSN 2278-8808 SJIF 2014 = 4.889
An International Peer Reviewed & Referred
SCHOLARLY RESEARCH JOURNAL
FOR INTERDISCIPLINARY STUDIES

CONTENT KNOWLEDGE AND PEDAGOGICAL CONTENT KNOWLEDGE IN THE EIGHTH GRADE MATHEMATICS TEXTBOOK OF WEST BENGAL BOARD OF SECONDARY EDUCATION

Subir Sen[1] & Tuhin Kumar Samanta[2]

Assistant Professor, B. Ed. Department, Kalna College, The University of Burdwan, West Bengal, India.

Associate Professor, Department of Education, the University of Burdwan, West Bengal, India

Abstract

Present work deals with an analysis of content knowledge and pedagogical content knowledge in eighth grade mathematics textbook of West Bengal Board of Secondary Education, West Bengal, and India. Concepts which are explored in the algebra together with necessary concepts which should be included are also discussed. An analysis on textual presentation and exercises supplied with necessary recommendations are argued.

Key words: *Mathematics textbook, Algebra, Content knowledge, Pedagogical Content Knowledge.*

Introduction

Shulman (1987) defines seven categories to provide a framework for teacher knowledge which are:

1. Content knowledge
2. General pedagogical knowledge eg classroom control, using group work
3. Pedagogical content knowledge
4. Curriculum knowledge
5. Knowledge of learners and their characteristics
6. Knowledge of educational contexts eg schools and the wider community
7. Knowledge of educational ends purposes and values

Shulman (1987) identified seven domains of teacher knowledge, one of which is pedagogical content knowledge. He explained why he identified pedagogical content knowledge as a knowledge domain for teachers as follows:

Pedagogical content knowledge is of special interest because it identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented and adapted to the diverse interests and abilities of learners, and presented for instruction.

Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue. (p. 8) Shulman claimed that pedagogical content knowledge is a distinct body of knowledge even though knowledge of content and knowledge of pedagogy contribute to it. He also noted that pedagogical content knowledge includes knowledge of learners, knowledge of educational context, and knowledge of instructional materials.

Tamir (1988) made a distinction between general pedagogical knowledge and subject-matter-specific pedagogical knowledge. He claimed that each type of knowledge is composed of four categories-namely, student, curriculum, instruction, and evaluation- but they have different meanings in each domain. He also identified teachers' skills in diagnosing students' conceptual difficulties in a given topic and their knowledge about effective use of instructional tools as subject-matter-specific pedagogical knowledge.

Ball and Bass (2000) identified teachers' knowledge of students' difficulties and appropriate teaching strategies to eliminate those difficulties as part of teachers' pedagogical content knowledge. They defined pedagogical content knowledge as follows:

'Pedagogical content knowledge is a special form of knowledge that bundles mathematical knowledge with knowledge of learners, learning, and pedagogy. These bundles offer a crucial resource for teaching mathematics, for they can help the teacher anticipate what students might have trouble learning, and have ready alternative models or explanations to mediate those difficulties (p. 88).'

Wang Wei Sönnerhed (2011) studied algebra textbook for CK and PCK and wrote 'The primary aim of the study is to explore what pedagogical content knowledge regarding solving quadratic equations that is embedded in mathematics textbooks. The secondary aim is to analyze the algebra content related to solving quadratic equations from the perspective of mathematics as a discipline in relation to algebra history. It is about what one can find in the textbook rather than how the textbook is used in the classroom (p-5).'

Sen and Samanta (2015) analyzed content knowledge and pedagogical content knowledge in sixth grade mathematics textbook of West Bengal Board of Secondary Education, West Bengal, India. According to them 'It seems that the algebra content of the text book demands slide modifications. Lastly it is suggested that there should be a concept summary listed at the end of the text and an exercise containing problems on every concept at the extreme end of the units discussed before for correlation and evaluation of concepts.'

Methodology

To explore the nature of Content Knowledge (CK) represented in the textbook and expected Pedagogical Content Knowledge (PCK) in the framework, it is necessary to discuss Van Dormolin's (1986) classification of teaching perspectives and learning perspectives of Schmidt et al (1997).

Based on their classification and the CK-PCK overall framework one may consider the following criteria for analyzing algebra content textual presentation as follows:

1. Consistency and clearness of Mathematical content: A mathematical text should be consistent and clear to the reader. "There must be no errors, either of computation or of logic. Proofs might be incomplete, but not false. Conventions must be used consistently. […] the content must be clear to the intended reader." (Van Dormolen, 1986, p. 151).
2. Mathematical theoretical aspects: This criterion concerns knowledge elements such as mathematical theorems, rules, definitions, methods and conventions. Such mathematical knowledge is called "kernels" (Van Dormolen, 1986, p. 146)

3. Mathematical content development and connections: This criterion is based on the classification of Schmidt et al. (1997). By means of this criterion, one may investigate how mathematical content topics relate to each other in the chapter of algebra. The aim is to explore the embedded teaching trajectory related to text.
4. Mathematical representations and applications: This category often reflects different views. A formalistic view regards mathematics as a set of concepts, rules, theorems and structures. Mathematics applications are often regarded as informal view. In an informal view students are encouraged to engage in activities like generalizing, classifying, formalizing, ordering, abstracting, exploring patterns and so on, and new ideas are encouraged (De Lange, 1996; Freudenthal, 1991; Goldin, 2008; Pepin et al., 2001; Van Dormolen, 1986; Vergnaud, 1987).
5. Language use: In which way are mathematical theorems, definitions, and rules explained and illustrated: formally in a mathematical language or pedagogically in combination with everyday language, in order to make sense for a student reader.
6. To analyze different kinds of mathematics exercises, activities and problems as well as tests in the textbook, it is important to analyzing mathematics tasks in the textbooks (Brändström, 2005). One may consider the following points:

A. Routine exercises refer to the kind of exercises that require students to use newly presented mathematical concepts, rules or algorithmic procedures illustrated in examples, in order to get familiar with the content. This kind of exercises is often at a basic level and requires simple and similar operations or reasoning to those just presented.

B. Exercises that require students to evaluate, analyze and reason mathematically instead of merely computing mechanically (Brändström, 2005). Such exercises intend to encourage students to understand the integration of mathematics concepts and procedures (Hiebert & Carpenter, 2007; Hiebert & Lefevre, 1986).

C. Exercises that are related to real world contexts. Such exercises are often word problems (or called real world problems) and the pedagogical reason of using them is to bring reality into the mathematics classroom, to create occasions for learning and practicing the different aspects of applied problem solving without the practical contact with the real world situation (Chapman, 2006). They reflect the view of

mathematics applications in real-life situations (De Lange, 1996; Freudenthal, 1991; Goldin, 2008; Pepin et al., 2001; Van Dormolen, 1986; Vergnaud, 1987).

Results and Discussions

Name of the eighth grade mathematics text book of West Bengal Board of Secondary Education is 'Ganitprava (class VIII)'. Following chapters are considered for discussions:

Chapter IV: Multiplication and division of Polynomial (pages 42-49)

Concepts which are explored in this chapter are:

1. Concept of polynomial is introduced with the help of 1, 2, 3, 4 terms by activity based approach.
2. Multiplication of two polynomials representing them as a side of a rectangle with the help of activity based approach is discussed.
3. Examples of such multiplications actually represent the area of the rectangle is illustrated.
4. The concept $A \times B = C$ implies $C \div B = A$ and $C \div A = B$ is represented by activity based method.
5. The concept of usual division is introduced without and with remainder.

Examples are well arranged and according to the concepts presented in the text.

To analyze the mathematical exercise represented in this unit it is observed that

1. Routine exercises are included.
2. Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.
3. Questions are represented according to the structure of the text and these are sequentially arranged.
4. Most of the problems are chosen from everyday life of the learner and common people.

Chapter V: Determination of Volume (Algebric formula with power 3) (pages 50-64)

Concepts which are explored is listed below:

1. Concept of perfect cube and which is not perfect cube is illustrated with the help of activity.
2. A table is constructed by natural numbers and their cubes.

3. Very interesting example of Hardy-Ramanujan number is illustrated ($1729 = 12^3 + 1^3 = 10^3 + 9^3$).

4. Concept of cube root is illustrated by factorization.

5. Concept of cube of monomial and binomial is illustrated.

6. An example of $(x+2)^3 = (x+2).(x+2)^2$ is done by multiplication.

7. The concept of $(a+b)^3 = (a+b).(a+b)^2$ is used to get $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ and $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$.

8. With the help of these two identities some examples are illustrated.

9. The generalization of $(x+y+z)^3$ with the help of $(a+b)^3$.

10. To get the identity $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$, $(a-b)^3 = [a+(-b)]^3$ is used in previous identity and rearranging them to get the identity $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$.

11. Some routine examples are illustrated.

12. Two identities $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ and $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ are established with the help of previous identities.

13. Using $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ and $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ some examples of factorization is illustrated.

Examples are well arranged and according to the concepts presented in the text.

Some suggestions for modification of text are as follows:

- Why cube is introduced is not clear. It seems that to relate the fact volume of a cube is equals to (side)^3. So a clear discussion of relationship between side and volume is necessary. It may be noted that the relationship among volume of parallelepiped and its length bradh and height plays a vital role in geometrical verification.
- Geometric verifications for the identities may be incorporated.

To analyze the mathematical exercise represented in this unit it is observed that

1. Routine exercises are included.

2. Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.

3. Questions are represented according to the structure of the text and these are sequentially arranged.

4. Most of the problems are chosen from everyday life of the learner and common people.

Chapter XIII: Factorization of Algebraic Expressions (pages 120-131)

Following concepts are represented:

1. Concept of area of square or rectangle is used to represent the algebraic expressions and their adjacent sides represent factors of the algebraic expressions by activity.

2. Some examples are illustrated with the help of the following identities

$a^2+2ab+b^2=(a+b)^2=(a+b)(a+b)$, $a^2-2ab+b^2=(a-b)^2=(a-b)(a-b)$,

$a^2-b^2=(a+b)(a-b)$, $a^3+b^3=(a+b)(a^2-ab+b^2)$ and $a^3-b^3=(a-b)(a^2+ab+b^2)$.

3. A list of identities is supplied.

4. The identity $x^2+(a+b)x+ab=(x+a)(x+b)$ and its use in factorization is illustrated.

5. Factorization of the algebraic expression $x^2+5x+4=(x+4)(x+1)$ represents a rectangle whose adjacent sides are $(x+4)$ and $(x+1)$ respectively is shown by activity method.

6. The concept of the identity $x^2+(a+b)x+ab=(x+a)(x+b)$ depends on the analysis of the coefficient of the middle term [i.e. (a+b)x], that is why it is called analysis of the middle term method of factorization is explored.

7. Some routine examples are illustrated

8. Concept of factorization of the algebraic expressions like mx^2+px+q is illustrated.

Examples are well arranged and according to the concepts presented in the text.

Slide rearrangement is suggested as:

- Examples 11 and 12 are actually overlapping but some discussions are necessary for the particular problem.
- It is suggested that example 15 should placed before example 14.

To analyze the mathematical exercise represented in this unit it is observed that

1. Routine exercises are included.

2. Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.

3. Questions are represented according to the structure of the text and these are sequentially arranged.

4. Most of the problems are chosen from everyday life of the learner and common people.

Chapter XIV: Highest Common Factor (HCF) and Least Common Multiplier (LCM) (pages 132-136)

Concepts which are explored is listed below:

1. Concept of HCF is introduced by activity. However the unit (meter) chosen is not practicable.
2. Scheme for calculating HCF is illustrated as
3. Routine examples of HCF are illustrated.
4. Concept of LCM is introduced by activity. However the unit (decimeter) chosen is not practicable.
5. Scheme for calculating LCM is illustrated as
6. Routine examples of HCF are illustrated.

Examples are well arranged and according to the concepts presented in the text. It is suggested that cyclic organization of factors should be discussed with examples. For example, a^2-b^2 and b^2-a^2 have the same factor $(a+b)(a-b)$ or $(a+b)(b-a)$ respectively.

To analyze the mathematical exercise represented in this unit it is observed that

1. Routine exercises are included.
2. Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.
3. Questions are represented according to the structure of the text and these are sequentially arranged.

Chapter XV: Simplification of Algebraic Expressions (pages 137-142)

Following concepts are explored:

1. An activity is introduced but unit (meter) of measurement is not practicable. It seems that similar type of activity is performed in the previous chapter. There is a doubt about appropriateness of this type of activity for this chapter.
2. Routine examples are illustrated.

Some suggestions are as follows:

- Activity which is introduced may represent $\frac{a}{4}$ or $\frac{x}{7}$ etc. but it does not approaches to simplification of algebraic expressions.

- This chapter is introduced mainly for the development of computational skills where use of identities and concepts of HCF and LCM is required.

To analyze the mathematical exercise represented in this unit it is observed that

1. Routine exercises are included.
2. Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.
3. Questions are represented according to the structure of the text and these are sequentially arranged.

Chapter XIX: Construction and Solution of Liner Equations (pages 183-192)

Concepts which are explored is listed below:

1. Construction of equation by activity is presented by with illustration.
2. Construction of equation whose root is known is presented.
3. Some routine examples are illustrated for solution of equations.

Examples are well arranged and according to the concepts presented in the text.

To analyze the mathematical exercise represented in this unit it is observed that

1. Routine exercises are included.
2. Exercises are arranged in such a way that can evaluate, analyze the concepts of the students. Also these exercises help to increase the power of mathematical reasoning of the learner.
3. Questions are represented according to the structure of the text and these are sequentially arranged.
4. Most of the problems are chosen from everyday life of the learner and common people.

Concluding remarks

It is mentioned in the text book that the book is written on the basis of NCF 2005 and learning will be activity based. It is actually psychological to logical approach to relate mathematics to real world. Teachers are encouraged to help in constructing students' knowledge. It seems that the algebra content of the text book demands slight modifications. Lastly it is suggested that there should be a concept summary listed at the end of the text and an exercise containing problems on every concept at the extreme end of the units for correlation and evaluation of concepts.

References

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple Perspectives on Mathematics Teaching and Learning*. Westport, CT : Ablex Publishing.

Brändström, A. (2005). *Differentiated tasks in mathematics textbooks: A analysis of the levels of difficulty*. Luleå : Luleå tekniska universitet.

Chapman, O. (2006). Classroom practices for context of mathematics word problems. *Educational Studies in Mathematics, 62*(2), 211-230.

De Lange, J. (1996). Using and applying mathematics in education. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds.), *International Handbook of Mathematics Education* (pp. 49-97). Dordrecht / Boston / London: Kluwer Academic Publishers.

Freudenthal, H. (1991). *Revisiting mathematics education: China lectures*. Dordrecht, NL: Kluwer Academic Publishers.

Goldin, G. (2008). Perspectives on representation in mathematical learning and problem solving. In L. D. English (Ed.), *Handbook of international research in mathematics education* (2 ed., pp. 176-201). New York: Routledge.

Hiebert, J., & Carpenter, T. P. (2007). Learning and teaching with understanding. In F. K. Lester Jr (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 65-97). Charlotte, NC: Information Age Pub.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum.

Pepin, B., Haggarty, L., & Keynes, M. (2001). Mathematics textbooks and their use in English, French and German classrooms: A way to understand teaching and learning cultures. *Zentralblatt für Didaktik der Mathematik, 33*(5), 158-175.

Schmidt, W. H., McKnight, C. C., Valverde, G. A., Houang, R. T., & Wiley, D. E. (1997). *Many visions, many aims - volume 1: A cross - national investigation of curricular intentions in school mathematics*. Boston: Kluwer Academic Publishers.

Sen, S., & Samanta, T. K. (2015). Content Knowledge and Pedagogical Content Knowledge in the sixth grade mathematics textbook of West Bengal Board of Secondary

Education. *International Journal of Multidisciplinary Educational Research*, Volume 4, Issue 5(3), 2015.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching *Educational Research, 15* (2), 4-14.

Tamir, P. (1988). Subject-matter and related pedagogical knowledge in teacher education. *Teaching and Teacher Education, 4*, 99–110.

Vergnaud, G. (1987). Conclusion. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 227-232). Hillsdale, NJ: Erlbaum.

Wang Wei Sönnerhed (2011). *An analysis of content knowledge and pedagogical content knowledge concerning algebra in mathematics textbooks in Swedish upper secondary education,* Göteborgs Universitet.